WEEKLY TEST RANKER'S BATCH-02 TEST - 1 Balliwala SOLUTION Date 08-09-2019

[PHYSICS]

1. According to Kepler's first law, every planet moves in an elliptical orbit with the sun situated at one of the foci of the ellipse.

   In options (a) and (b) sun is not at a focus while in (c) the planet is not in orbit around the sun. Only (d) represents the possible orbit for a planet.

2. Kepler's law $T^2 \propto R^3$

3. In arrangement 1, both forces act in the same direction. In arrangement 3, both the forces act in opposite direction. This alone decides in favour of option (a),

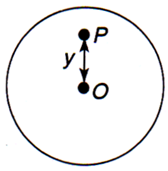
4. Time period of a revolution of a planet,

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{\frac{GM_S}{r}}} = \frac{2\pi r^{3/2}}{\sqrt{GM_S}}$$

5. Gravitational force is independent of the medium. Thus, gravitational force will be same i.e., F.

6. If a point mass is placed inside a uniform spherical shell, the gravitational force on the point mass is zero. Hence, the gravitational force exerted by the shell on the point mass is zero.

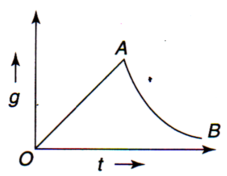
7. $g_d = g\left(1 - \frac{d}{g}\right)$



or $\quad g_d = g\frac{R-d}{R}$

or $\quad g_d = \frac{gy}{R}$ or $g_d \propto y$

So, within the Earth, the acceleration due to gravity varies linearly, with the distance from the centre of the Earth. This explains the linear portion OA of the graphs.



8. The value of g at the height h from the surface of earth

$$g' = g\left(1 - \frac{2h}{R}\right)$$

The value of g at depth x below the surface of earth

$$g' = g\left(1 - \frac{x}{R}\right)$$

These two are given equal, hence $\left(1 - \frac{2h}{R}\right) = \left(1 - \frac{x}{R}\right)$

On solving, we get $x = 2h$

9. Acceleration due to gravity $g = \frac{4}{3}\pi\rho GR \quad \therefore\ g \propto \rho R$

or $\frac{g_m}{g_e} = \frac{\rho_m}{\rho_e} \cdot \frac{R_m}{R_e}$

$$\left[\text{As } \frac{g_m}{g_e} = \frac{1}{6} \text{ and } \frac{\rho_e}{\rho_m} = \frac{5}{3} \text{(given)}\right]$$

$$\therefore \quad \frac{R_m}{R_e} = \left(\frac{g_m}{g_e}\right)\left(\frac{\rho_e}{\rho_m}\right) = \frac{1}{6} \times \frac{5}{3} \therefore R_m = \frac{5}{18} R_e$$

10. Acceleration due to gravity $g = \frac{GM}{R^2}$

$$\therefore \quad \frac{g_{\text{moon}}}{g_{\text{earth}}} = \frac{M_{\text{moon}}}{M_{\text{earth}}} \cdot \frac{R_{\text{earth}}^2}{R_{\text{moon}}^2} = \left(\frac{1}{80}\right)\left(\frac{4}{1}\right)^2$$

$$g_{\text{moon}} = g_{\text{earth}} \times \frac{16}{80} = \frac{g}{5}$$

11. Acceleration due to gravity $g = \frac{4}{3}\pi\rho GR$

$$\therefore \quad g_1 : g_2 = R_1\rho_1 : R_2\rho_2$$

12. We know $g = \frac{GM}{R^2} = \frac{GM}{(D/2)^2} = \frac{4GM}{D^2}$

If mass of the planet $= M_0$ and diameter of the planet $= D_0$. Then $g = \frac{4GM_0}{D_0^2}$.

13. $\frac{g'}{g} = \left(\frac{R}{R+h}\right)^2 = \left(\frac{R}{R+2R}\right)^2 = \frac{1}{9} \quad \therefore g' = \frac{g}{9}$.

14. Acceleration due to gravity on earth is

$$g = \frac{GM_E}{R_E^2} \qquad \text{(i)}$$

As $\quad \rho = \frac{M_E}{\frac{4}{3}\pi R_E^3} \Rightarrow M_E = \rho\frac{4}{3}\pi R_E^3$

Substituting this value in Eq. (i), we get

$$g = \frac{G\left(\rho\frac{4}{3}\pi R_E^3\right)}{R_E^2} = \frac{4}{3}\pi\rho G R_E \text{ or } \rho = \frac{3g}{4\pi G R_E}$$

15. $$\Delta U = U_2 - U_1 = \frac{mgh}{1+\frac{h}{R_e}} = \frac{mgR_e}{1+\frac{R_e}{R_e}} = \frac{mgR_e}{2}$$

$$\Rightarrow \quad U_2 - (-mgR_e) = \frac{mgR_e}{2} \Rightarrow U_2 = -\frac{1}{2}mgR_e$$

16. Potential energy $U = \frac{-GMm}{r} = -\frac{GMm}{R+h}$

$$U_{\text{initial}} = -\frac{GMm}{3R} \text{ and } U_{\text{final}} = -\frac{-GMm}{2R}$$

Loss in PE = gain in $KE = \frac{GMm}{2R} - \frac{GMm}{3R} = \frac{GMm}{6R}$

17. $$\Delta u = -\left(\frac{GMm}{R+h}\right) - \left(-\frac{GMm}{R}\right)$$

$H = 3R$

$$\Delta u = -\frac{GMm}{4R} + \frac{GMm}{R} = \frac{3GMm}{4R^2} \times R$$

$$\Delta u = \frac{3}{4}mgR$$

18. $U = \frac{-GMm}{r}$, $K = \frac{GMm}{2r}$ and $E = \frac{-GMm}{2r}$

For a satellite U, K and E vary with r and also U and E remain negative whereas K remains always positive.

19. $$v = \sqrt{\frac{GM}{R+h}}$$

For first satellite $h = 0$, $v_1 = \sqrt{\frac{GM}{R}}$

For second satellite $h = \frac{R}{2}$, $v_2 = \sqrt{\frac{2GM}{3R}}$

$$v_2 = \sqrt{\frac{2}{3}}v_1 = \sqrt{\frac{2}{3}}v$$

20. Orbital velocity of the satellite is

$v=\sqrt{\frac{GM_E}{r}}$ where M_E is the mass of the earth

Kinetic energy, $K=\frac{1}{2}mv^2=\frac{GM_E m}{2r}$

where m is the mass of the satellite.

$$K \propto \frac{1}{r}$$

Hence, option (b) in incorrect.

Linear momentum, $p=mv=m\sqrt{\frac{GM_E}{r}}$

$$p \propto \frac{1}{\sqrt{r}}$$

Hence, option (c) is incorrect.

Frequency of revolution, $\nu=\frac{1}{T}=\frac{1}{2\pi}\sqrt{\frac{GM_E}{r^3}}$

$$\nu \propto \frac{1}{r^{3/2}}$$

Hence, option (d) is correct.

21. Time period, $T=\frac{2\pi R}{\sqrt{\frac{GM_m}{R}}}=\frac{2\pi R^{3/2}}{\sqrt{GMm}}$

where the symbols have their meanings as given. Squaring both sides, we get

$$T^2=\frac{4\pi^2 R^3}{GMm}$$

22. Total energy of the orbiting satellite of mass m having orbital radius r is

$E=-\frac{GMm}{2r}$ where M is the mass of the planet.

Additional kinetic energy required to transfer the satellite from a circular orbit of radius R_1 to another radius R_2 is

$$=E_2-E_1$$

$$=-\frac{GMm}{2R_E}-\left(-\frac{GMm}{2R_1}\right)=-\frac{GMm}{2R_2}+\frac{GMm}{2R_1}$$

$$=\frac{GMm}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

23. $\frac{1}{2}mv_{\min}^2 = \left[-\frac{GMm}{r} - \frac{GMm}{r}\right]$

$$-\left[-\frac{GMm}{(2r-a)} - \frac{GMm}{a}\right]$$

$$= \frac{2GMm(a^2 - 2ar + r^2)}{ar(2r-a)}$$

or $v_{\min} = \sqrt{\frac{GM}{a}} \times \frac{2(r-a)}{[r(2r-a)]^{1/2}}$

So, $K = \frac{2(r-a)}{[r(2r-a)]^{1/2}}$

24. $v = \sqrt{\frac{GM}{r}}$ if $r_1 > r_2$ then $v_1 < v_2$

Orbital speed of satellite does not depend upon the mass of the satellite.

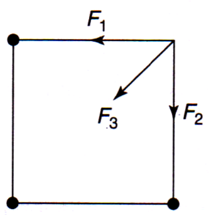
25.

If two particles of mass m are placed x distance apart then force of attraction $\frac{Gmm}{x^2} = F$ (Let)

26.

$F_1 = F_2 = \frac{GM}{a^2}$

Resultant of F_1 and F_2 is $\sqrt{2}\,\frac{GM}{a^2}$



Now, $F_3 = \frac{GM}{(\sqrt{2}\,a)^2} \frac{GM}{2a^2}$

Now, $\frac{\sqrt{2}\,GM}{a^2}$ and $\frac{GM}{2a^2}$ act in the same direction.

Their resultant is $\frac{\sqrt{2}\,GM}{a^2} + \frac{GM}{2a^2}$ or $\frac{GM}{a^2}\left[\sqrt{2} + \frac{1}{2}\right]$

$$G\left[M - \frac{10}{100}M\right]$$

27.

Time period does not depend upon the mass of satellite, it only depends upon the orbital radius.

According to Kepler's law

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{r}{2r}\right)^{3/2} = \frac{1}{2\sqrt{2}}$$

28. Since the planet is at the centre, the focus and centre of the elliptical path coincide and the elliptical path becomes circular and the major axis is nothing but the diameter. For a circular path:

$$\frac{mv^2}{r} = \sqrt{\frac{GM}{r^2}}m$$

Also $T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\sqrt{GM}} \Rightarrow T^2 = \frac{4\pi^2 r^3}{GM}$

$\Rightarrow \quad r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} = \text{Radius}$

$\Rightarrow \quad \text{Diameter (major axis)} = 2\left(\frac{GMT^2}{4\pi^2}\right)^{1/3}$

29.

Acceleration due to gravity $g = \frac{GM}{R^2}$

$$\therefore \quad \frac{g_{moon}}{g_{earth}} = \frac{M_{moon}}{M_{earth}} \cdot \frac{R^2_{earth}}{R^2_{moon}} = \left(\frac{1}{80}\right)\left(\frac{4}{1}\right)^2$$

$$g_{moon} = g_{earth} \times \frac{16}{80} = \frac{g}{5}$$

30.

Radius of earth $R = 6400$ km $\therefore \ h = \frac{R}{4}$

Acceleration due to gravity at a height h

$$g_h = g\left(\frac{R}{R+h}\right)^2 = g\left(\frac{R}{R+\frac{R}{4}}\right)^2 = \frac{16}{25}g$$

At depth 'd' value of acceleration due to gravity

$$g_d = \frac{1}{2}g_h \text{ (According to problem)}$$

$$\Rightarrow \quad g_d = \frac{1}{2}\left(\frac{16}{25}\right)g \Rightarrow g\left(1-\frac{d}{R}\right) = \frac{1}{2}\left(\frac{16}{25}\right)g$$

By solving we get $d = 4.3 \times 10^6$ m

31.

Acceleration due to gravity $g = \frac{GM}{R^2}$

$$\therefore \quad \frac{g_{\text{planet}}}{g_{\text{earth}}} = \frac{M_{\text{planet}}}{M_{\text{earth}}}\left(\frac{R_{\text{earth}}}{R_{\text{planet}}}\right)^2 = \frac{1}{10} \times \left(\frac{3}{1}\right)^2 = \frac{9}{10}$$

If a stone is thrown with velocity u from the surface of the planet then maximum height

$$H = \frac{u^2}{2g} \Rightarrow \frac{H_{\text{planet}}}{H_{\text{earth}}} = \frac{g_{\text{earth}}}{g_{\text{planet}}}$$

$$\Rightarrow \quad H_{\text{planet}} = \frac{10}{9} \times H_{\text{earth}} = \frac{10}{9} \times 90 = 100 \text{ metre}$$

32.

Let m be mass of a body.

$\therefore$ Weight of the body on the surface of the earth is

$$W = mg = 250 \text{ N}$$

Acceleration due to gravity at a depth d below the surface of the earth is

$$g' = g\left(1 - \frac{d}{R_E}\right)$$

Weight of the body at depth d is

$$W' = mg' = mg\left(1 - \frac{d}{R_E}\right)$$

Here, $d = \frac{R_E}{2}$

$$\therefore \quad W' = mg\left(1 - \frac{R_E/2}{R_E}\right) = \frac{mg}{2} = \frac{W}{2} = \frac{250 \text{ N}}{2} = 125 \text{ N}$$

33.

Acceleration due to gravity at a place of latitude λ due to the rotation of earth is

$$g' = g - R_E\omega^2 \cos^2 \lambda$$

At equator, $\lambda = 0°$, $\cos 0° = 1$

$$\therefore \quad g' = g_e = g - R_g\omega^2$$

At poles, $\lambda = 90°$, $\cos 90° = 0$

$$\therefore \quad g' = g_P = g$$

$$\therefore \quad g_P - g_e = g - (g - R_E\omega^2) = R_E\omega^2$$

34.

$$g' = g\left(1 - \frac{d}{R}\right) \Rightarrow \frac{g}{4} = g\left(1 - \frac{d}{R}\right) \Rightarrow d = \frac{3R}{4}$$

35.

Potential energy of the body at a distance $4R_e$ from the surface of earth

$$U = -\frac{mgR_e}{1 + h/R_e} = -\frac{mgR_e}{1+4} = -\frac{mgR_e}{5}$$

[As $h = 4R_e$ (given)]

So minimum energy required to escape the body will be

$$\frac{mgR_e}{5}.$$

36. Total energy of orbiting satellite at a height h is

$$E = -\frac{GM_E m}{2(R_E + h)}$$

The total energy of the satellite at infinity is zero.

$\therefore$ Energy expended to rocket the satellite out of the earth's gravitational field is

$$\Delta E = E_\infty - E$$

$$= 0 - \left(-\frac{GM_E m}{2(R_E + h)}\right) = \frac{GM_E m}{2(R_E + h)}$$

37.

$$\Delta K.E. = \Delta U$$

$$\frac{1}{2}MV^2 = GM_e M\left(\frac{1}{R} - \frac{1}{R+h}\right) \qquad \text{(i)}$$

Also $g = \frac{GM_e}{R^2}$ (ii)

On solving (i) and (ii) $h = \frac{R}{\left(\frac{2gR}{V^2} - 1\right)}$

38.

$$\Delta u = -\left(\frac{GMm}{R+h}\right) - \left(-\frac{GMm}{R}\right)$$

$$H = 3R$$

$$\Delta u = -\frac{GMm}{4R} + \frac{GMm}{R} = \frac{3GMm}{4R^2} \times R$$

$$\Delta u = \frac{3}{4}mgR$$

39.

40.

$$v \propto \frac{1}{\sqrt{r}}.$$

% increase in speed = $\frac{1}{2}$ (% decrease in radius)

$$= \frac{1}{2}(1\%) = 0.5\%$$

i.e., speed will increase by 0.5%

41. **Potential energy = 2 (total energy) = $2E_0$**

42.

Interstellar velocity $v' = \sqrt{\frac{GM}{r}} = R\sqrt{\frac{g}{(R+h)}}$

$= \sqrt{v^2 - v_e^2}$

where v = projection velocity

$$\frac{R^2 g}{(r+h)} = v^2 - 2gR \text{ Solving } v^2 = \frac{23gR}{11}$$

43.

Angular speed of earth = angular speed of geostationary satellite

$$T \propto \frac{1}{\omega}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{\omega_1}{\omega_2} = \frac{1}{2} \Rightarrow T_2 = \frac{T_1}{2}$$

$$\therefore \quad \left(\frac{r_2}{r_1}\right)^{3/2} = \frac{T_2}{T_1} = \frac{1}{2}$$

$$\frac{r_2}{r_1} = \left(\frac{1}{2}\right)^{2/3} = \frac{1}{4^{1/3}}$$

$$r_2 = \frac{r_1}{4^{1/3}}$$

44.

$$g = \frac{GM}{R^2} = G\frac{4}{3}\pi R^3 \cdot \rho$$

$$g = \frac{4}{3}G\pi R\rho \Rightarrow g \times R\rho$$

$$g' \times R'\rho' \Rightarrow \rho' = 2\rho$$

Given, $\frac{g}{g'} = 1$

$$\frac{R}{R_1} = 2 \Rightarrow R' = \frac{R}{2}$$

45.

According to the question, the gravitational force between the planet and the star is $F \propto \frac{1}{R^{5/2}}$

$$\therefore \quad F = \frac{GMm}{R^{5/2}}$$

where M and m be mass of star and planet respectively.

For motion of a planet in a circular orbit,

$$mR\omega^2 = \frac{GMm}{R^{5/2}}$$